RMSMC
Redmond Middle School Math Club

2223M
2022-2023 MATHCOUNTS Exam

Wednesday, January 11, 2023

INSTRUCTIONS

1. DO NOT BEGIN THIS EXAM UNTIL YOUR PROCTOR TELLS YOU.
2. This is a thirty question SHORT ANSWER test. All answers must be recorded in the correct location on the separate answer sheet.
3. SCORING: You will receive 1 point for each correct answer, 0 points for each problem left unanswered, and 0 points for each incorrect answer. Ties will be broken for top placement positions based on the highest numbered question answered correctly. If students are still tied, the process is repeated for the remainder of questions in reverse order. Exact ties will be broken at the sole discretion of the Math Club chair.
4. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
5. Figures are not necessarily drawn to scale.
6. Units are not necessary unless the question asks for time, where AM or PM should be specified.
7. Give all answers in simplest form, rationalizing the denominator if necessary. If you get a fractional answer, express it as a common fraction unless otherwise indicated. If the answer is dealing with money, then round to the nearest hundredth.
8. Please make sure to write your name where indicated.
9. When your proctor gives the signal, begin working on the problems. You will have 40 minutes to finish your exam.
10. When you finish the exam, please go over your answers again to check your work.

Questions for this exam were authored by Anish Deshpande, Joseph Kaim, Layla Kaim, Evan Kim, Sarah Wen, and Iris Zheng.

ANSWER SHEET

Name		Score 1	Score 2	Final
Grade		Initial 1	Initial 2	

Do not write in shaded regions.

	Answer	1 or 0	1 or 0
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
1-15 Total			

	Answer	1 or 0	1 or 0
16			
17			
18			
19			
20			
21			
22			
23			
24			
25			
26			
27			
28			
29			
30			
16-30 Total			

1. Let n be the sum $1^2 + 2^2 + 3^2 + \cdots + 100^2$. What is the remainder when n is divided by 7?

2. On the planet Rith, they use a different form of currency. On Rith, 7 $Icks$ are worth 23 $Bicks$ and 3 $Bicks$ and worth 14 $Micks$. How many $Micks$ are 21 $Icks$ worth?

3. Eric combines 4 liters of a lemonade that is 30% sugar and 2 liters of a lemonade that is 15% sugar. Then then drinks a liter of his new mixture and decides that it needs more sugar. How many liters of sugar must he add for the mixture to be 40% sugar?

4. Compute $_7C_1 + {}_8C_2 + {}_9C_3 + {}_{10}C_4 + {}_{11}C_5$.

5. The sum of the first n positive even integers is 4290. What is n?

6. My box of favorite writing utensils has a 3:5 ratio of pens to pencils. If Amanda steals 9 pencils but leaves 3 pens in the box as an apology, I will have an equal number of pens and pencils. How many pencils do I have after the exchange?

7. If $0.\overline{9119}$ is written as a fraction $\frac{a}{b}$ with a and b being positive integers with a greatest common divisor of 1, what is $b - a$?

8. When one corner of a 1-foot by 9-inch sheet of paper is folded onto the diagonally opposite corner, a crease is formed. The length, in inches, can be represented as $\frac{a}{b}$. What is $\sqrt{a + b}$?

9. The largest three-digit integer whose digits are distinct and form a geometric sequence can be written as ABC. What is CBA?

10. A sphere is inscribed inside a hemisphere of radius 6. What is the volume of this sphere?

11. Currently, carbon dioxide is the most dominant greenhouse gas – through the burning of fossil fuels, industrial production, and land use change (Our World in Data). Around 50 billion tons of CO2 are emitted by the technology, industrial, residential, and commercial sectors annually, which is 40% higher than the emissions in 1990. However, there are other gases contributing to climate change: methane, nitrous oxide, etc. Methane is especially dangerous due to the faster resonating of waves and the higher heat trapping ability; this is why workers flare natural gas in the North Dakota Bakken Shale after hydraulic fracturing for oil. Overall, 600 million tons of methane are released into the atmosphere each year. What were the carbon emissions in 1990 rounded to the nearest billion? Please give your answer using scientific notation.

12. In an equilateral triangle with side length of 2 units, three circles with radius of 1 unit are drawn centered at each of the vertices. What is the area of the triangle that is not contained within any of the circles?

13. A regular hexagon is inscribed in a circle with a diameter of 24 inches. What is the area of the circle outside of the hexagon?

14. What is 2023_4 written in base 10?

15. Amy shoots an arrow at a target with four rings each centered at the same point. The diameter of the innermost circle is 2 feet, and each ring's diameter after that increases by 2 feet. If Amy successfully hits the target, what is the probability, as a decimal, it lands on either the innermost ring or the outermost ring?

16. What is the absolute difference of the smaller angle between the hour hand and minute hand at 2:46 and 6:42?

17. $\triangle ABC$ has coordinates of $A(-4, 2)$, $B(6, 8)$, and $C(2, -8)$. What is the coordinate where the angle bisector of $\angle A$ intersects with $\underline{BC}$?

18. Suppose you have a quadratic of the form $f(x) = x^2 + kx + 36$ with integer roots. What is the maximum possible sum of the roots of f?

19. Jane has $20 dollars of allowance to distribute to her 4 children. To make sure no one is unhappy, she must give each child at least one dollar. In how many ways can Jane distribute her allowance?

20. The surface areas of 3 faces of a rectangular prism are 42, 91, and 78. What is the volume of the rectangular prism?

21. Farmer John raises 3-legged cows and 5-legged chicken on his farm. If he counts 55 legs and 15 animals, how many 3-legged cows does he have?

22. List all real roots of the polynomial $1 + 2x + 3x^2 + 4x^3 + 3x^4 + 2x^5 + x^6$.

23. How many ways are there to rearrange the letters in the word COMBINATION?

24. Refer to the image below for this prompt. Suppose a given triangle has sides a, b, and c, with lengths 3, 4, and 5, respectively. $\overline{d}$ is drawn between the point of intersection of sides a and b and side c. What is the length of $\overline{d}$?
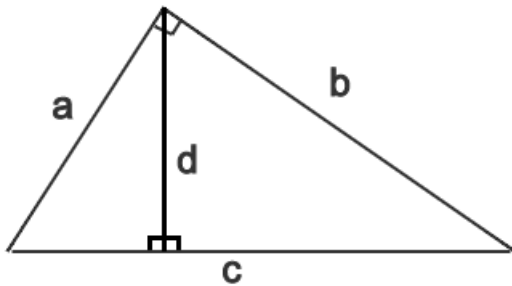
25. Jeff wants to drive from Redmond to Bellevue and back. To do so, he takes SR-520. The way there is a breeze, taking him only 10 minutes! Unfortunately, due to major accidents, he was only able to average 20 mph on the way back and was detoured 5 additional miles. If it took him 49 minutes to make this round trip, how far did he drive? Round your answer to the nearest mile.

26. At Redmond High School, children are only allowed to play three sports: football, soccer, and sportsball. Of the 2,000 kids in attendance at the school, 1,469 only play one sport, 419 play football and soccer, 331 kids play football and sportsball, 315 play all three sports, and 1,000 kids play sportsball. If everyone is required to play at least one sport, how many kids only play sportsball?

27. Jack and Jill are working together to empty a lake. Jack can empty the lake on his own in 5 hours, while Jill would need 6 hours to empty the lake. A malicious man by the name of Marguillo is working to keep the lake full, and he could fill an empty lake in 4 hours. How many minutes will it take for Jack and Jill to empty the lake, assuming that it is completely full at time 0?

28. Josephine has a fair coin, a fair four-sided die, and a fair six-sided die. She first flips the coin. If the coin shows heads, she rolls the four-sided dice. If the coin shows tails, she rolls the six-sided dice. What is the expected value that shows on the dice?

29. The fraction $\frac{25}{28}$ can be written in the form of $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$. What is $a + b + c$?

30. A quadratic function $f(x)$ exists such that $f(0) = 3$, $f(1) = -2$, and $f(2) = 7$. What is $f(3)$?